Assignment: Section 5.3: 2 (a,b), 8, 24, 26a, 28a, 32a  ( 7th edition)

**2.** Find *f (*1*)*, *f (*2*)*, *f (*3*)*, *f (*4*)*, and *f (*5*)* if *f (n)* is defined recursively by *f (*0*)* = 3 and for *n* = 0*,* 1*,* 2*, . . .*

**a)** *f (n* + 1*)* = −2*f (n)*.

**-6, 12, -24, 48, -96**

**b)** *f (n* + 1*)* = 3*f (n)* + 7.

**16, 55, 172, 523, 1576**

**8.** Give a recursive definition of the sequence {*an*}, *n* =

1*,* 2*,* 3*, . . .* if

**a)** *an* = 4*n* − 2.

**Basis Step: A(1) = 4 - 2 = 2**

**Recursive Step: A(n+1) = 4(n+1) - 2**

**= 4n + 4 - 2**

**= 4n - 2 + 4**

**= an + 4**

**b)** *an* = 1 + *(*−1*)n*.

**Basis Step: A(1) = 1+(-1)1 = 0**

**Recursive Step: A(n+1) = 1 + (-1)n+1**

**= 1 + -1n ( -11)**

**= 1 + (((-1)n + 1) -1) -1))**

**=1 + (an - 1)(-1)**

**=2 - an**

**c)** *an* = *n(n* + 1*)*.

**Basis Step: A(1) = 1(1+1) = 2**

**Recursive Step: A(n + 1) = n+1(n+1 + 1)**

**= n (n+1) + n + n+1 + 1**

**= an 2n + 2**

**d)** *an* = *n*2.

**Basis Step: A(1) = (1)2 = 1**

**Recursive Step: A(n+1) = (n + 1)2**

**= n2+2n+1**

**=an + 2n + 1**

**24.** Give a recursive definition of

**a)** the set of odd positive integers.

**Basis Step: 1∈ S**

**Recursive Step: If x ∈ S, then x + 2 is ∈ S**

**b)** the set of positive integer powers of 3.

**Basis Step: 3 ∈ S**

**Recursive Step: If x is ∈ S, then x3 ∈ S**

**c)** the set of polynomials with integer coefficients.

**Basis Step: 0 ∈ S**

**Recursive Step: If p(x) ∈ S, then p(x) + cxn ∈ S, where c ∈ Z, n ∈ Z, and n >= 0**

**26.** Let *S* be the subset of the set of ordered pairs of integers defined recursively by

*Basis step: (*0*,* 0*)* ∈ *S*.

*Recursive step:* If *(a, b)* ∈ *S*, then *(a* + 2*, b* + 3*)* ∈ *S* and *(a* + 3*, b* + 2*)* ∈ *S*.

**a)** List the elements of *S* produced by the first five applications of the recursive definition.

**(2, 3) (3, 2)**

**(4, 6) (6, 4) (5, 5)**

**(6, 9) (7, 8) (8, 7) (9, 6)**

**(8, 12) (9, 11) (10, 10) (11, 9) (12, 8)**

**(10, 15) (11, 14) (12, 13) (13, 12) (14, 11) (15,10)**

**28.** Give a recursive definition of each of these sets of ordered pairs of positive integers. [*Hint:* Plot the points in the set in the plane and look for lines containing points in the set.]

**a)** *S* = {*(a, b)* | *a* ∈ **Z**+*, b* ∈ **Z**+*,* and *a* + *b* is odd}

**Basis Step: a ∈ Z**+ **and b ∈ Z**+

**(a(1) + b(1)) + 2 is odd**

**Recursive Step: if (a,b) ∈ Z+ then (a + 1, b + 1) is odd, (a+2,b) is odd, (a ,b + 2) is odd**

**These conditions are all true if a + b is odd. If the sum of a and b is 3, which is the smallest case where a + b is odd and positive integers, then a =1 b =2, making them fitting of set S. The sum of (a+b) + 2 is 5 at its least, and (a -2, b) (a, b-2) (a-1, b-1) would have to have positive integers whose sum is odd and smaller than a+b, and therefore must be in S, and an application of the first recursive step would show (a, b) ∈ S**

**32. a)** Give a recursive definition of the function *ones(s)*, which counts the number of ones in a bit string s (A bit string is a string of zeros and ones).

Let Σ = {0.1)

Basis Step: ones(λ) = 0 (empty string w/ no 1's or 0's)

Recursive Step: If x **∈** Σ, and w **∈** Σ\*, then ones(wx) = ones(w) + x, where x is either 0 or 1.